Computational and Fourier Optics Mini-Course

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For some interesting examples see:
http://jansky.gsfc.nasa.gov/OSCAR

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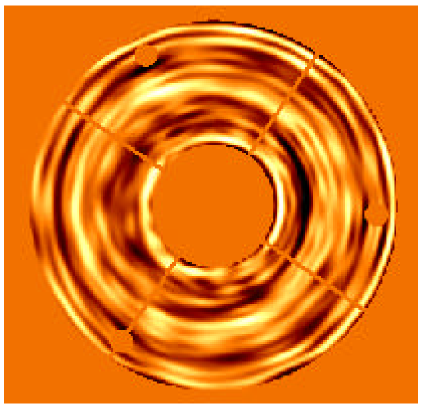
The Future of Space Imaging

Higher Resolution <=> larger Apertures

1990 | 2008 | > 2015

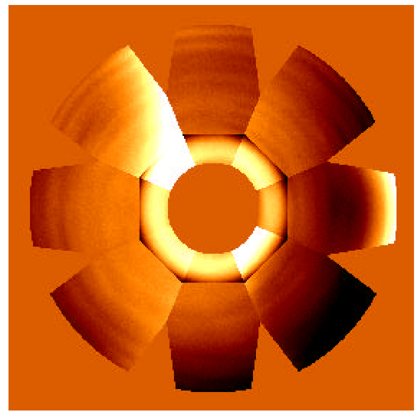
2.4 meter



Monolithic Aperture Systems

- Computer Complexity
  - direct image
  - flat fielding, calibration, registration
- Hubble Space Telescope, SIRTF, GOES etc...

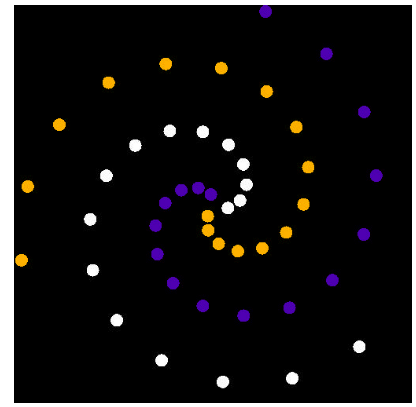
8 meter



Segmented Aperture Systems

- Computer Complexity
  - direct image
  - req's hardware/software control loop - segm't align (on-board vs on-ground ?)
  - In-situ image quality
  - Image Restoration ?
- NGST

> 20 meter



Interferometric/Sparse Aperture Systems

- Computer Complexity
  - No direct image
  - req's hardware/softw loop
  - on-board vs on-gnd ?
  - Restoration required
- Space Interferometer mission
- SPECS
- Stellar Imager
- Terrestial Planet Finder

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Computational Optics

- What is Computational Optics ?
  - Combine optics/imaging/systems with information sciences.
  - End-to-End Analytical/Computational simulation of entire sensor system:
    - Spectral Scene => Telescope/Interferometer => Instrument => Detector
    - Output should "mirror" actual system.
  - Design/Development of methods for "optimal information extraction".
    - Imaging/inteferometric/hyperspectral/coronagraphic etc.
    - Maximum Entropy/Maximum Likelihood etc.
- Applications of Computational Optics:
  - Systems concept studies; Systems and Instrument Design.
  - Modeling/Simulation & Performance Assessment.
  - WFS & Optical Control Systems.
  - Guidance/Navigation and Control.
  - Ground & Image Processing.
  - Optimal Algorithm Design and Validation.
- Large Demand => Develop as Core Competency

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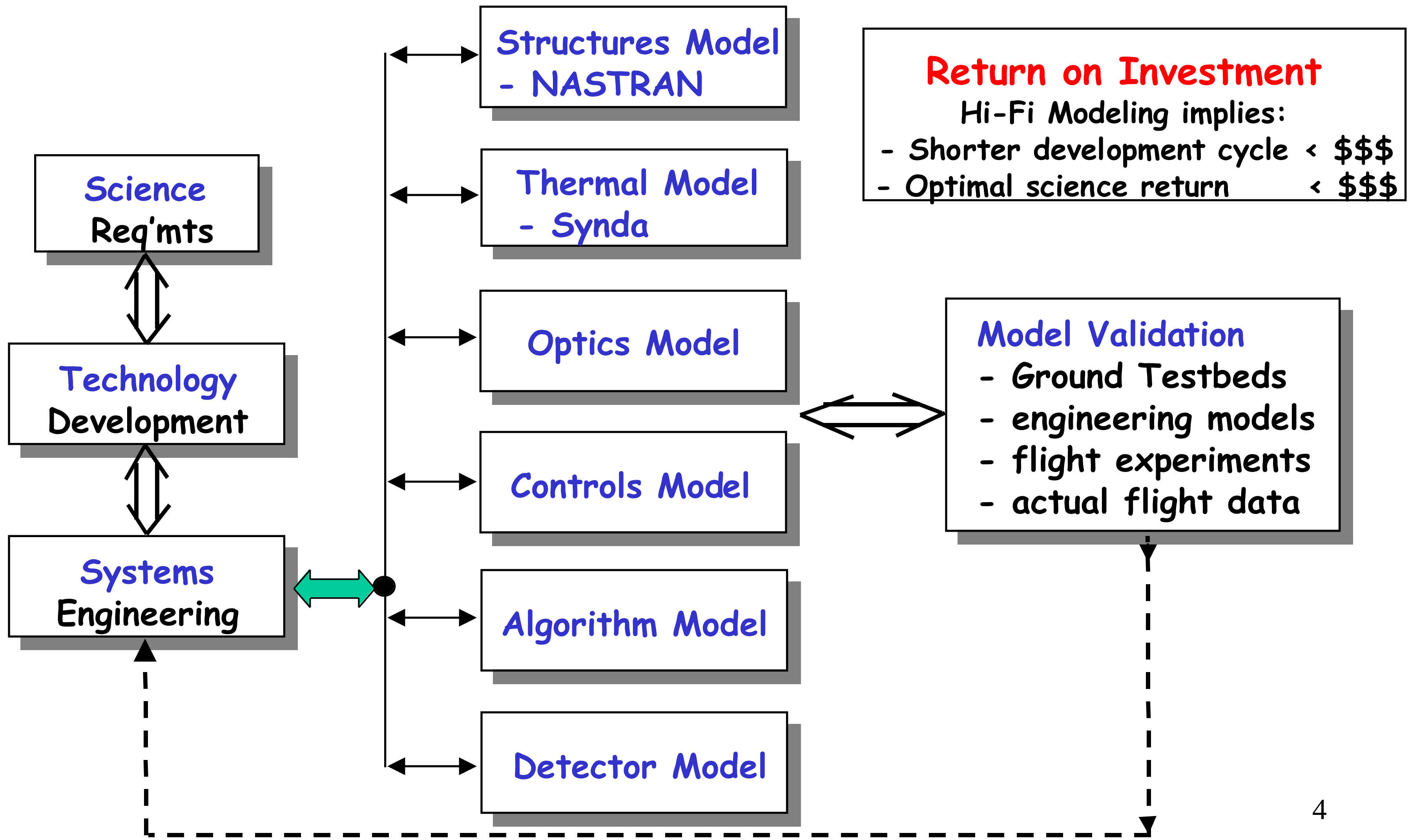
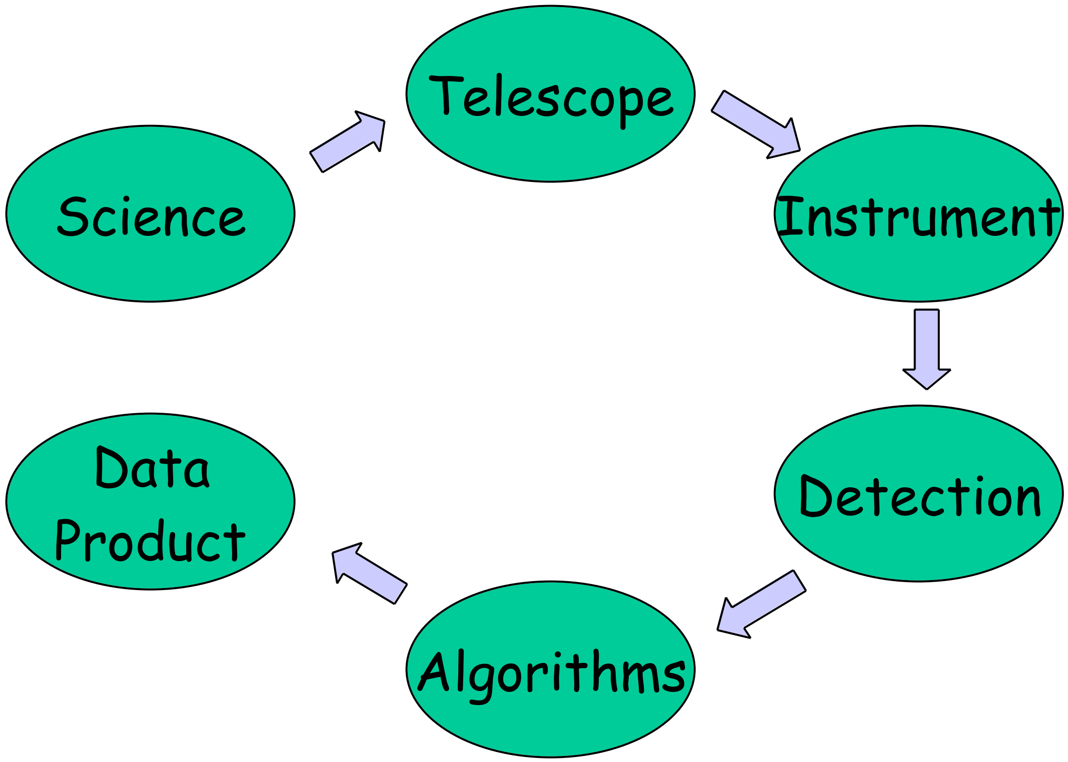
Modeling & Simulation

NASA Design Paradigm ?



Advanced Sensors and Systems

End-to-End Simulation <=> End-to-End optimal system design



Course Overview.

- Computational and Fourier Optics can be fun and enlightening. A little understanding will go a long way towards developing physical insight into this intrinsically mathematical disicpline.
- Attempt to impart intuition and insight with minimal mathematics by using only 3 concepts for imaging systems:

- What is Amplitude, Phase and Diffraction ?

  Definition of a "wave" and what happens as it propagates.
- What are Pupils and Focii ?

  Complex optical systems can be reduced to a simple ones
  Entrance pupil, exit pupil and focal plane.
  Relations via spatial Fourier Transforms.
- What is an Image ?

  Intuitively everyone knows what an image is, but what is it really ?
  Can you describe the image quality in useful terms ? Imaging systems are actually low pass filters. Effect of sampling, quantization and noise. Rules of thumb; when and when not to apply them.

These topics will be liberally discussed and emphasized via many images. Mathematics will be shown when necessary, however, it's use will not be the primary emphasis of this course. As time permits we will also delve into why Goddard needs Fourier Optics and some special topics.

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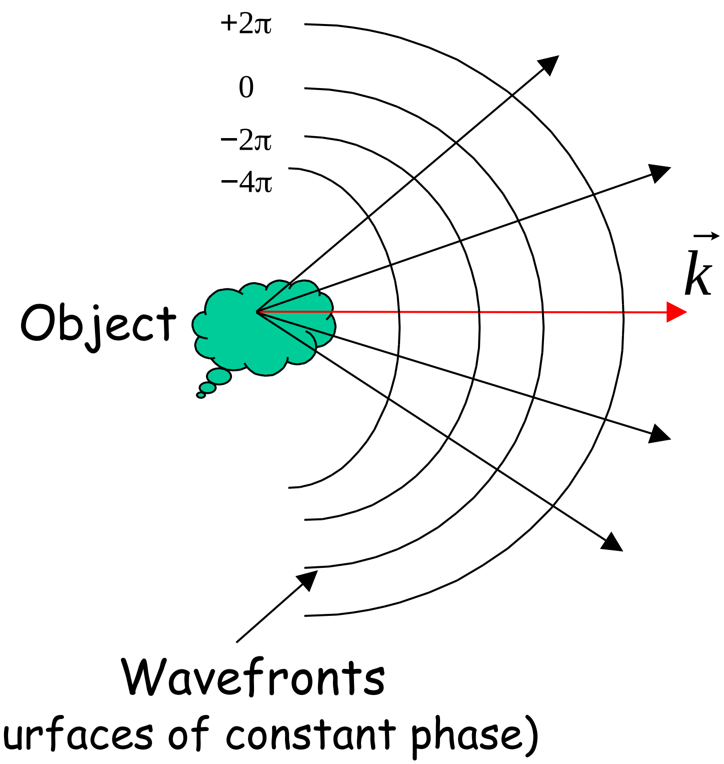
Amplitude and Phase



- Any object is a collection of points
- Each point emits a spherical wave

$$G(\vec{r}-\vec{r}\,') = \frac{e^{i\vec{k}\,(\vec{r}-\vec{r}\,')}}{|\vec{r}-\vec{r}\,'|}$$

- Solution of Helmholtz Equation

$${}^2E(\vec{r}) + k^2\varepsilon E(\vec{r}) = 0$$

- Each wave is independent of all others
  => Incoherent <=
- Surfaces of constant amplitude are parallel to surfaces of constant phase
  => Homogeneous wave <=
- Rays perpendicular to surfaces of constant

Amplitude and Phase

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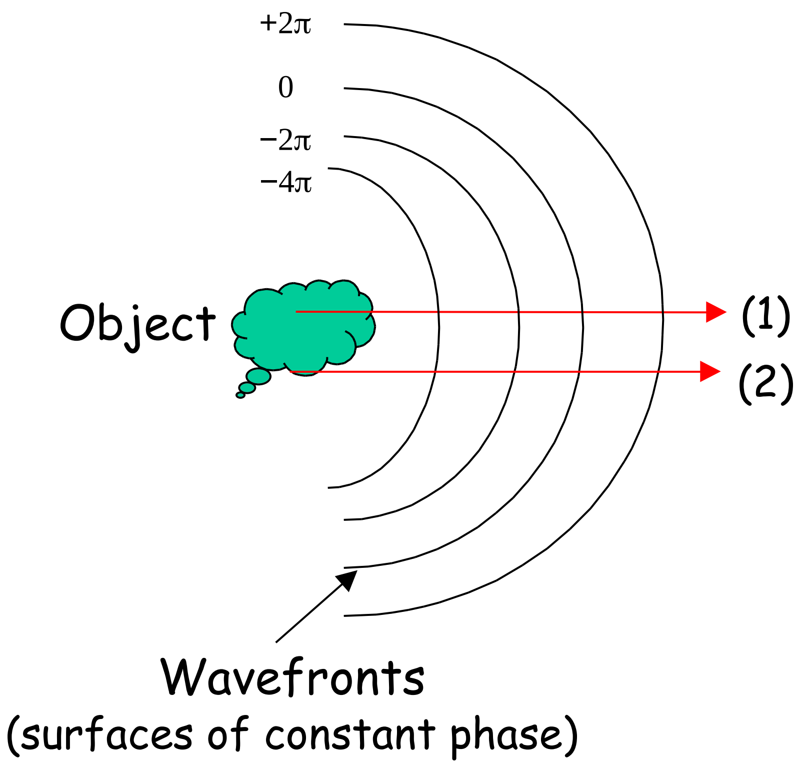

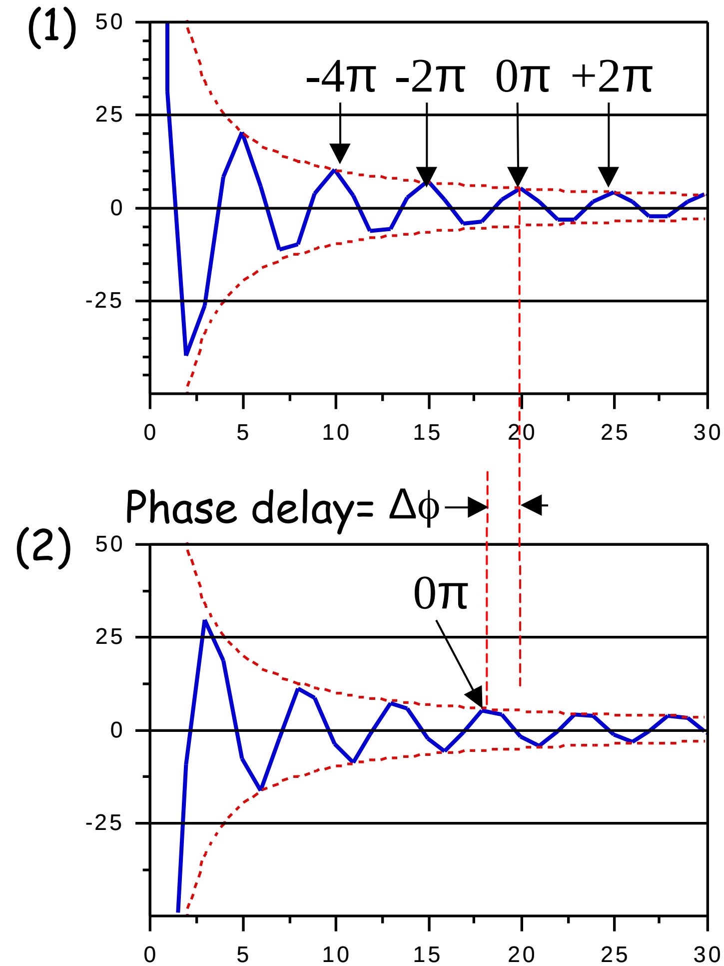

- Amplitude is magnitude of envelope
- Intensity (#photons) Amplitude^2
- Phase $\left(\Delta\phi = c\Delta t = \frac{2\pi}{\lambda}\Delta z\right)$ relative to a reference

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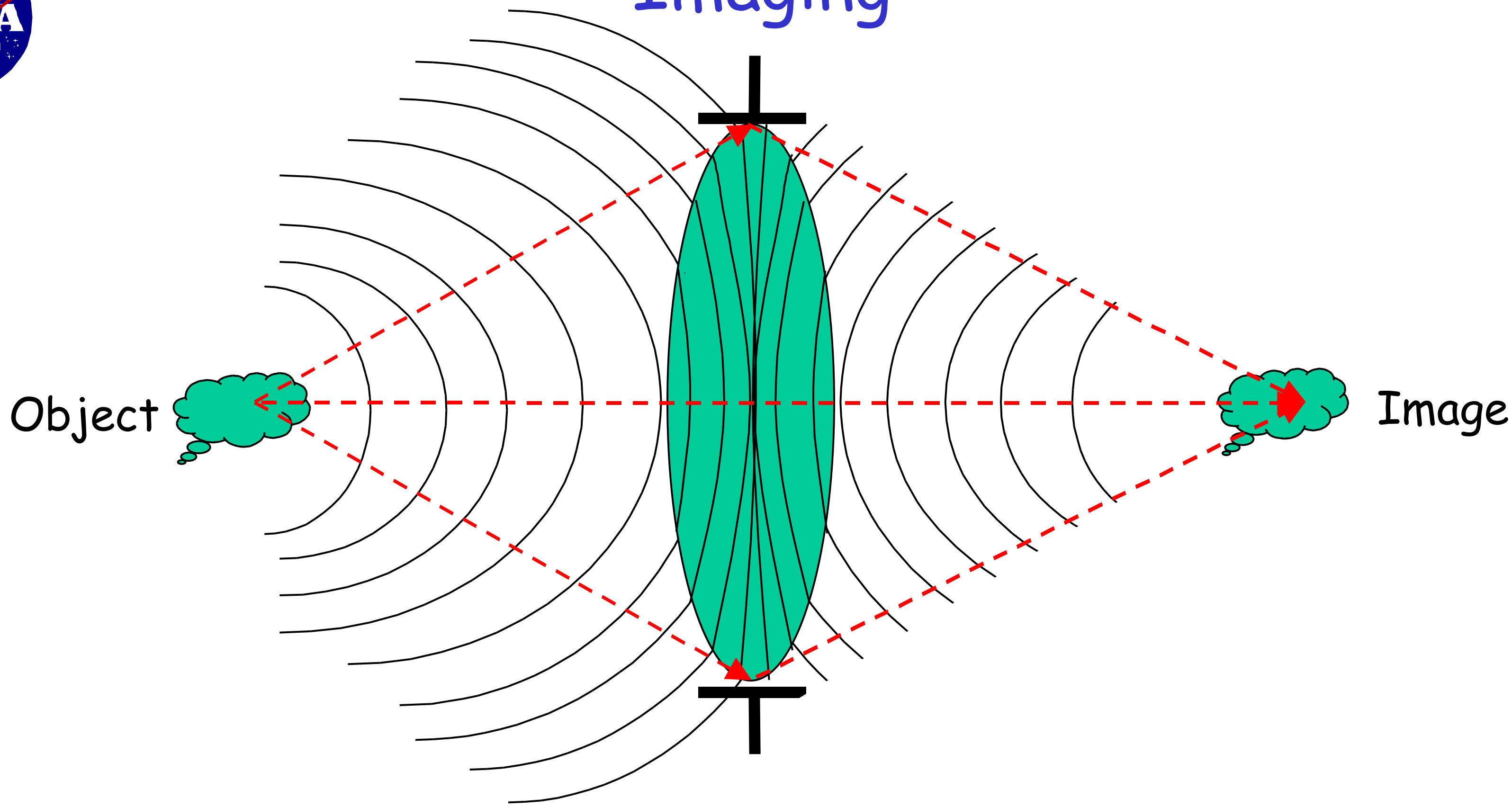
Imaging



- Speed of light slower within lens, c'=c/n
- Phase delay within lens reverses curvature
- Each object point ideally imaged to a point ? NO !
    - -Edges of lens => Diffraction
    - -Deformations/Misalignments => Aberrations

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Diffraction

- Truncated wavefront causes spreading of converging beam
- Each object point is "spread" into a point spread function (PSF)
- PSF determined from diffraction theory
- Solution of Helmholtz equation w/boundary conditions

$$\nabla^2 E(\vec{r}) + k^2 \varepsilon E(\vec{r}) = 0$$

- For focussing system PSF is given by:

$$PSF(x,y;\lambda) = \frac{1}{\lambda^2 F^2} \left| \iint_A A(u,v)\, e^{i\phi(u,v)}\, e^{-i2\pi\left(\frac{xu}{\lambda F} + \frac{yv}{\lambda F}\right)} du dv \right|^2$$

- 2D spatial Fourier Transform of the complex Pupil Function

$$P(u,v) = A(u,v)\, e^{ikW(u,v)}$$

- A(u,v) is the amplitude and W(u,v) is the phase delay

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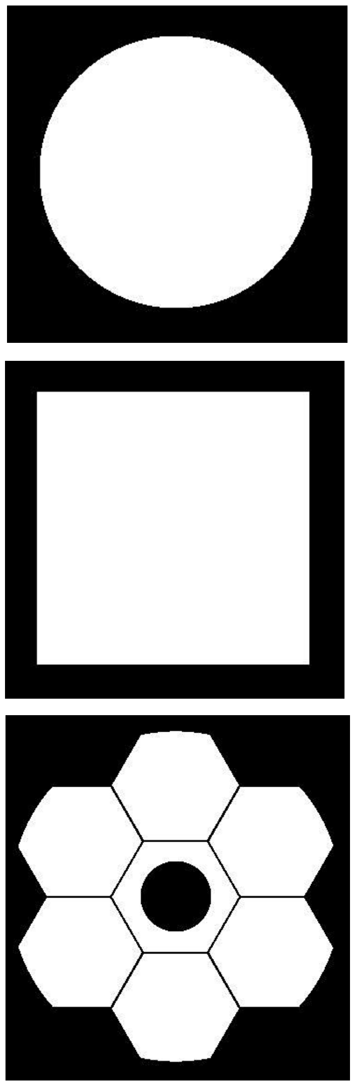
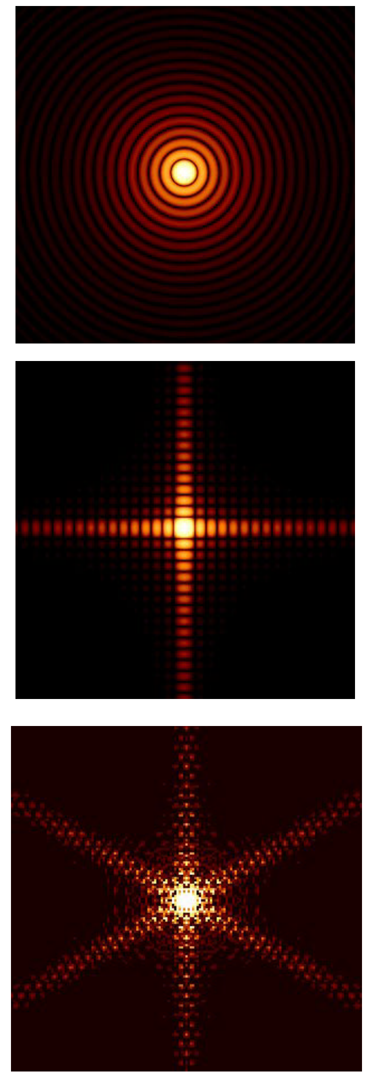
Pupils and PSFs

- Pupils and PSFs are related by Fourier Transforms
- Numerically implemented by Fast Fourier Transform (FFT)
- Edges give diffraction flairs

Pupils





PSFs

The Famous Airy Disk

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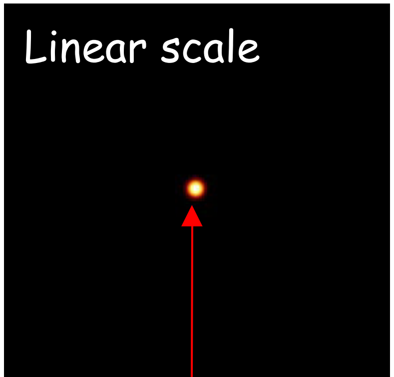
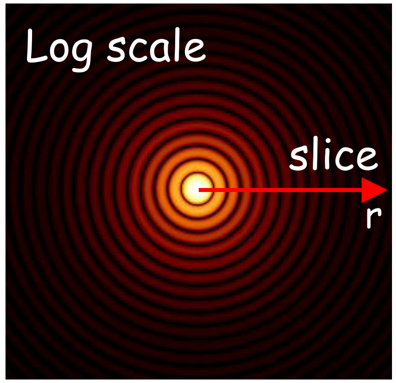
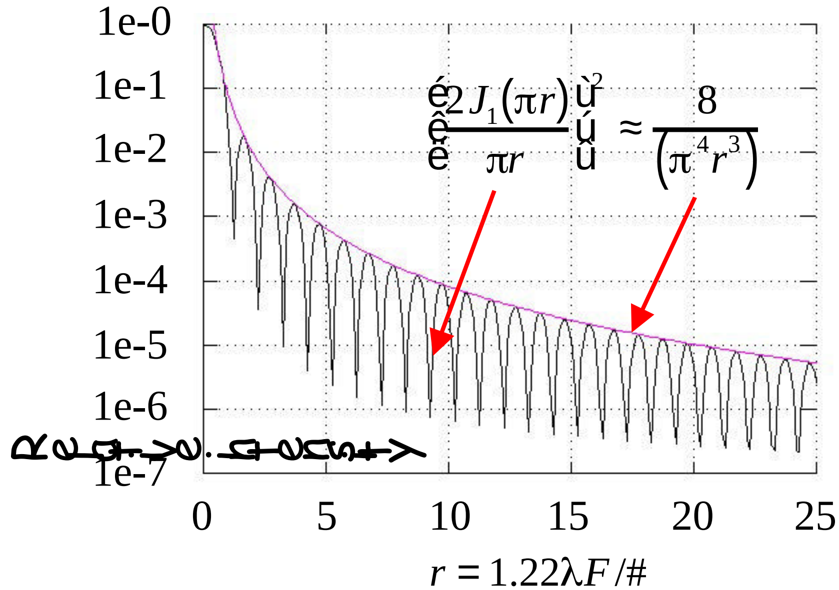
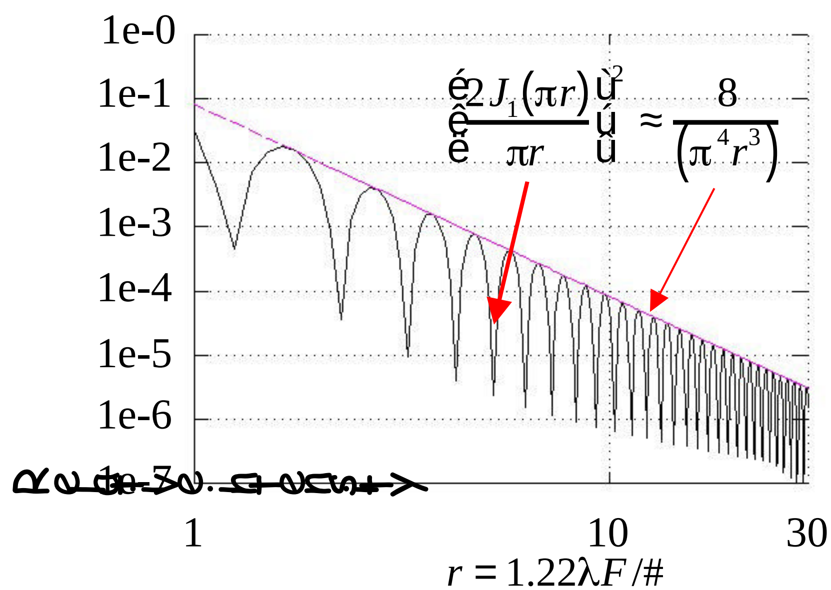




Core width
$\Delta r = 2.44\lambda F/\#$
zero to zero





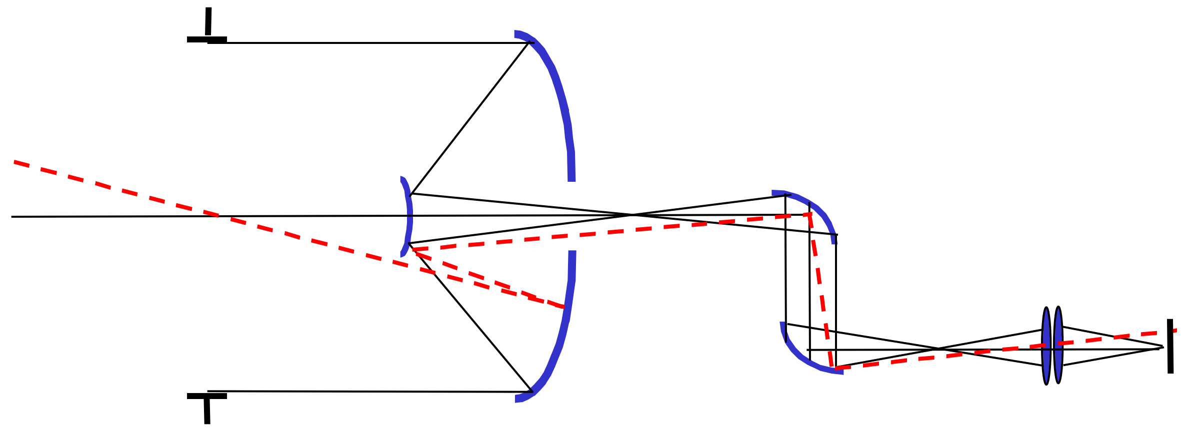
- Resolution

  focal plane => sky => ground

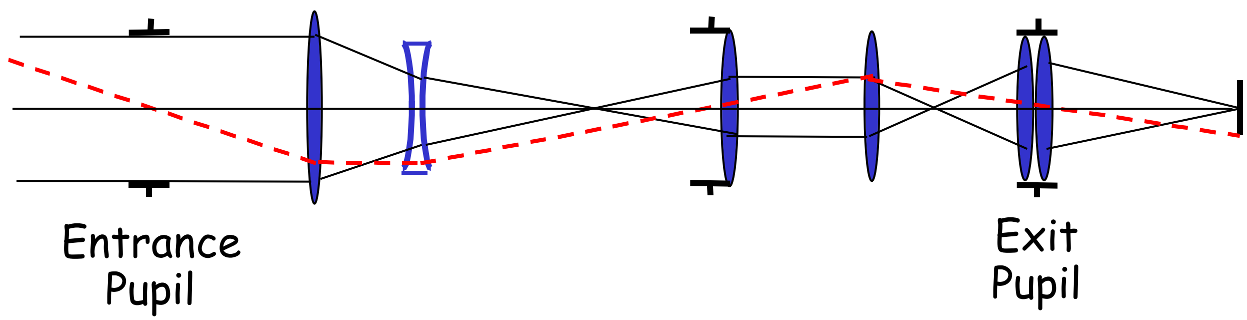
  2.44λF/# => 2.44λ/D => 2.44(λ/D)h
- PSF "falls off" like ~1/r^3
- "Diffraction Limited" PSF

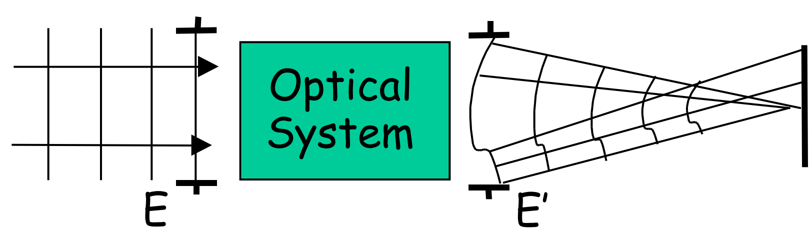
What happens with Complex Optical System ?

Answer: reduce to simple system via pupils & focii



- Unfold System
- ID Entrance Pupil
- Image of E = Exit Pupil
- Reduce system to EE'
- Easy Analysis





What can go wrong ? aka Aberrations

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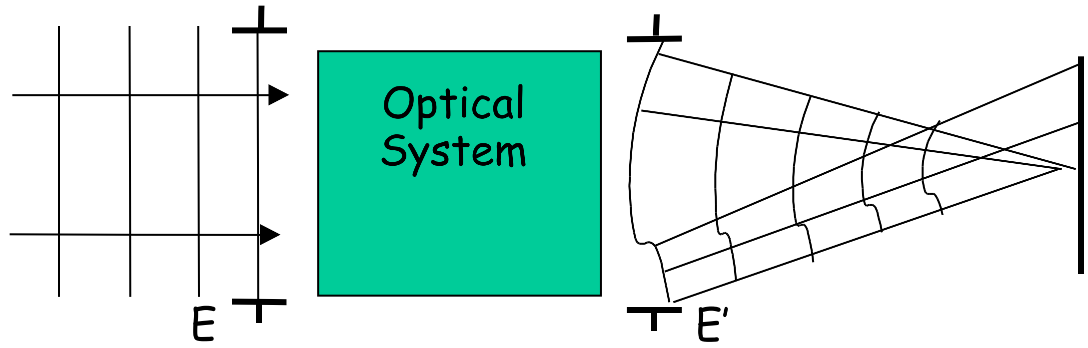

- "Pieces" of same wavefront don't arrive at same time
- Different optical path lengths for each ray => Aberrations
- Causes:
  - Design residuals
  - Misalignments (thermal/structural)
  - Deformations (thermal/structural)
  - Manufacturing errors (polishing etc.)
- Ray deviations are proportional to wavefront slope
- How to determine Aberrations:
  - Raytrace a bundle of rays from entrance to exit pupil
  - Calculate path length along each ray, remove mean OPL
  - phase delay is proportional to OPD
  - Insert phase delay (wavefront into diffraction integral)

Examples of Aberrated Wavefronts and PSFs

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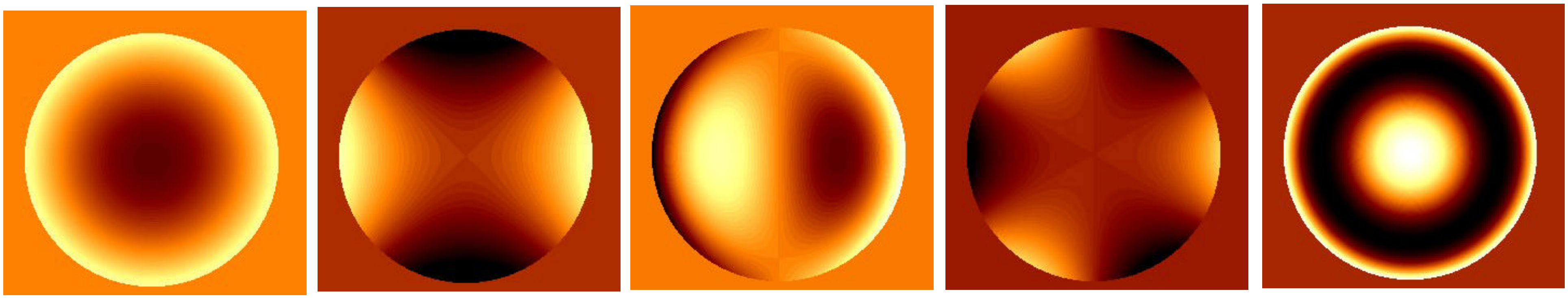
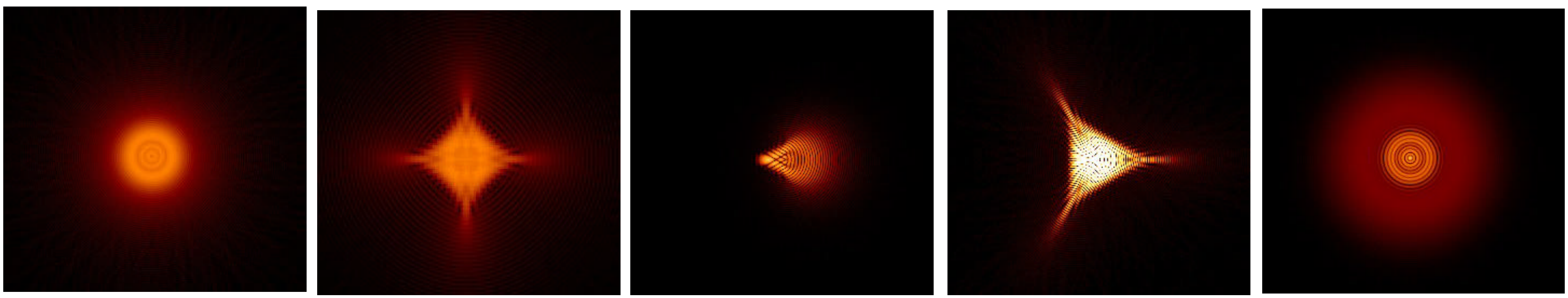
Wavefronts => Deviations from reference wavefronts in exit pupil



Focus | Astigmatism | Coma | Trefoil | Spherical



PSF => Impulse response in focal plane

Frequency Response and Transfer Function

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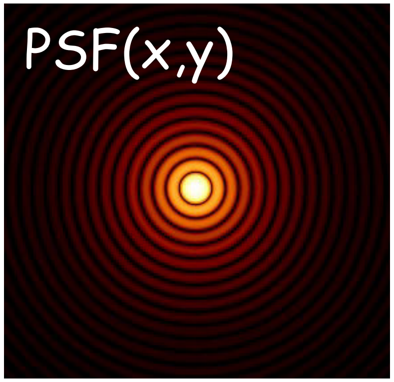
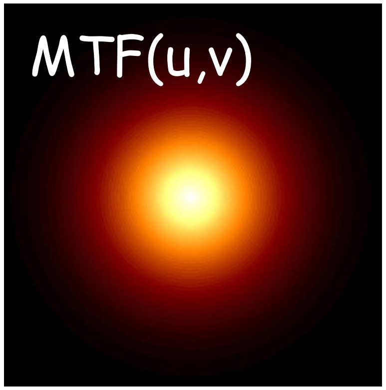
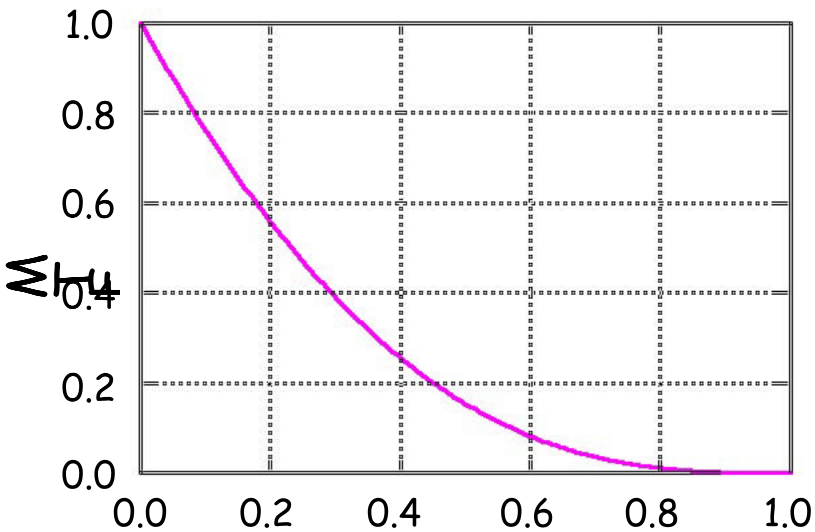
- PSF is "impulse response"
- FT{PSF} is Optical Transfer Function

Pupil P(u,v)	PSF(x,y)
P(u,v)*P(u,v)	OTF(u,v) MTF(u,v)=|OTF|

F.T. (Pupil → PSF)
F.T. (PSF → OTF)
= (P(u,v)*P(u,v) = OTF(u,v))







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Imaging

- Significance of PSF:
  - PSF gives image quality
  - PSF gives frequency response
  - PSF gives resolution
  - PSF used to specify design !
  - PSF is the "spatial impulse response"

- Image is given by 2D convolution of PSF with object

$$I(x,y) = \iint PSF(x-x', y-y')\, O(x',y')\, dx'dy'$$

$$\mathbf{d} = \mathbf{PO} + \mathbf{n}$$

$$d_j = \sum_k P_{jk} O_k$$

- Imaging is a 2D low-pass spatial filter !

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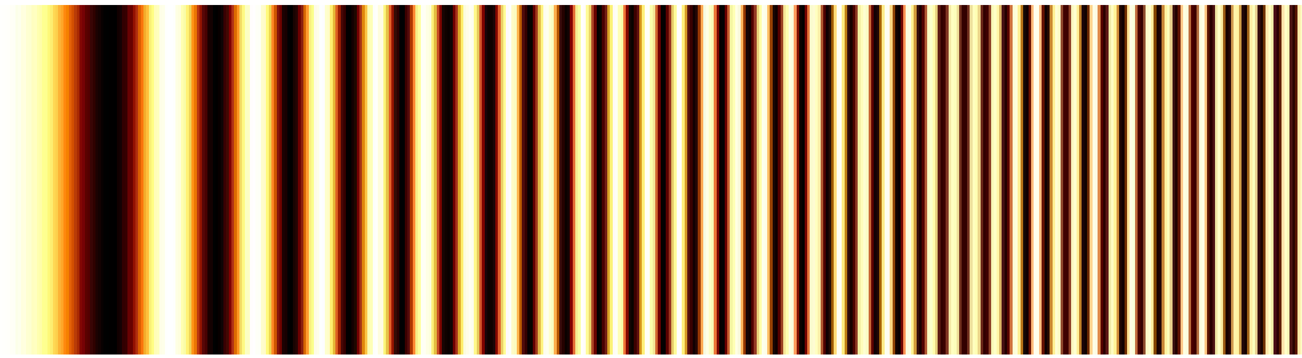
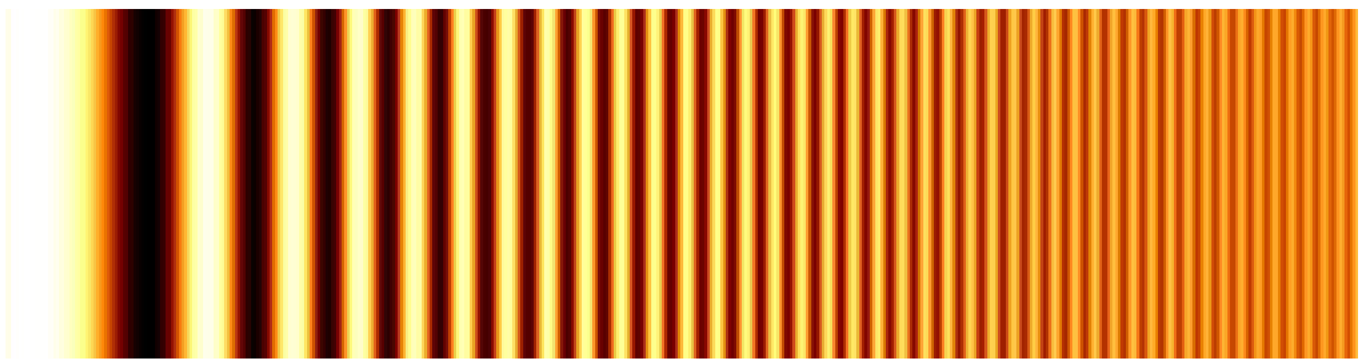
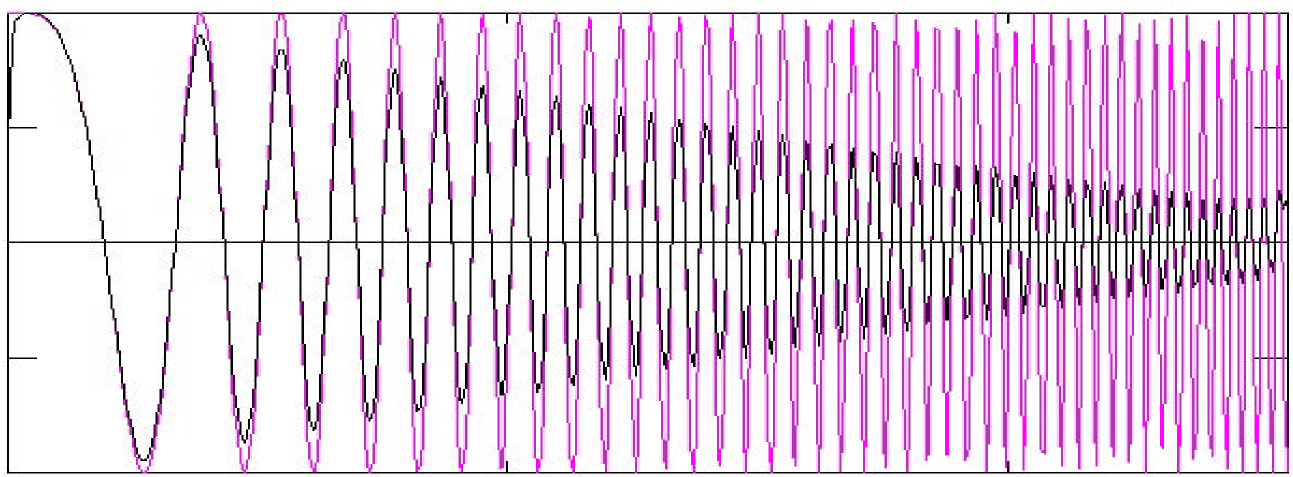
Chirped Pattern

Object



Image

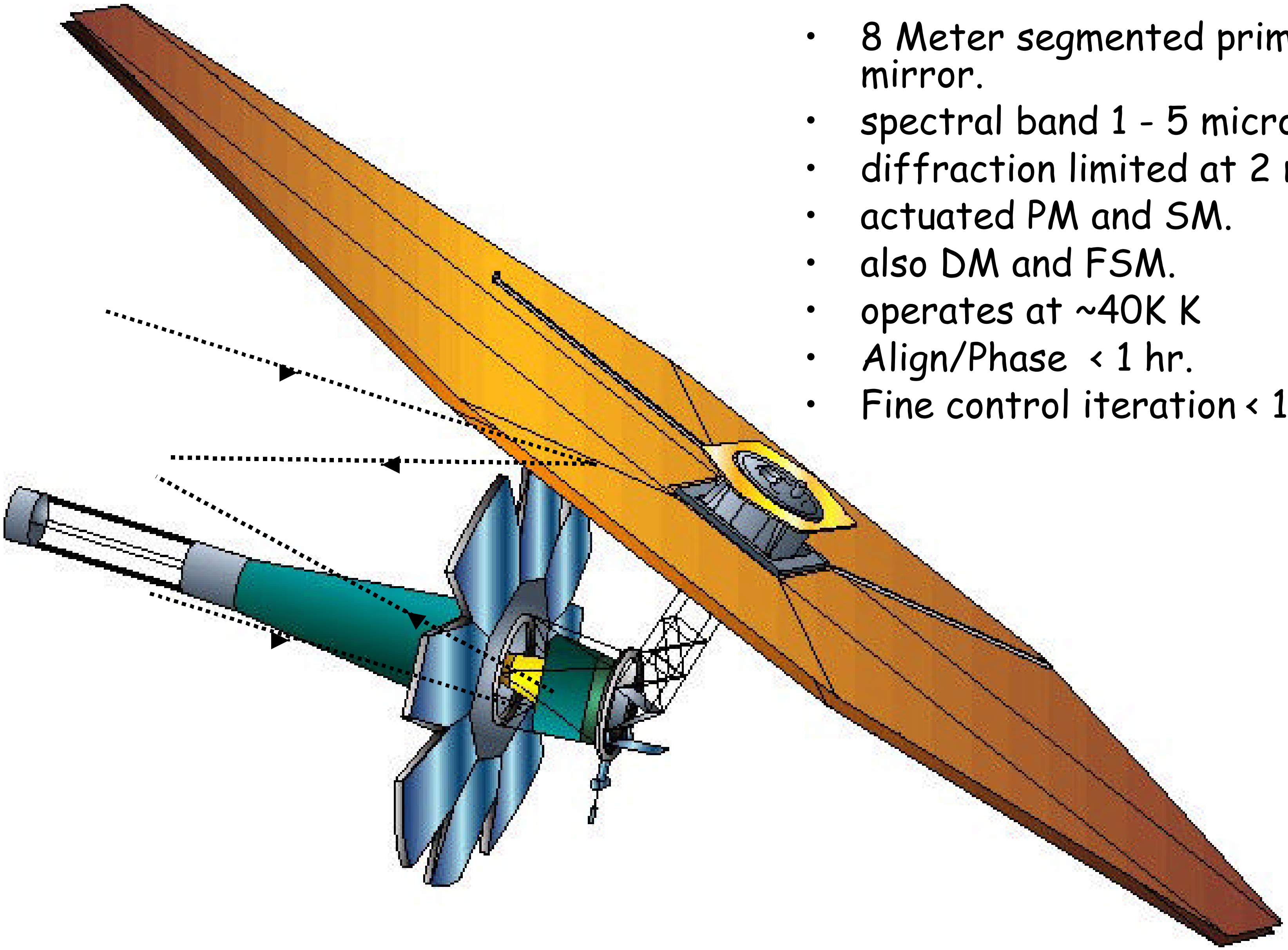




Some Examples of Computational Optics

- Next Generation Space Telescope
- Coronagraph - Direct Planetary Imaging

NGST GSFC/JPL Design



- 8 Meter segmented primary mirror.
- spectral band 1 - 5 micron.
- diffraction limited at 2 micron.
- actuated PM and SM.
- also DM and FSM.
- operates at ~40K K
- Align/Phase < 1 hr.
- Fine control iteration < 10 min.

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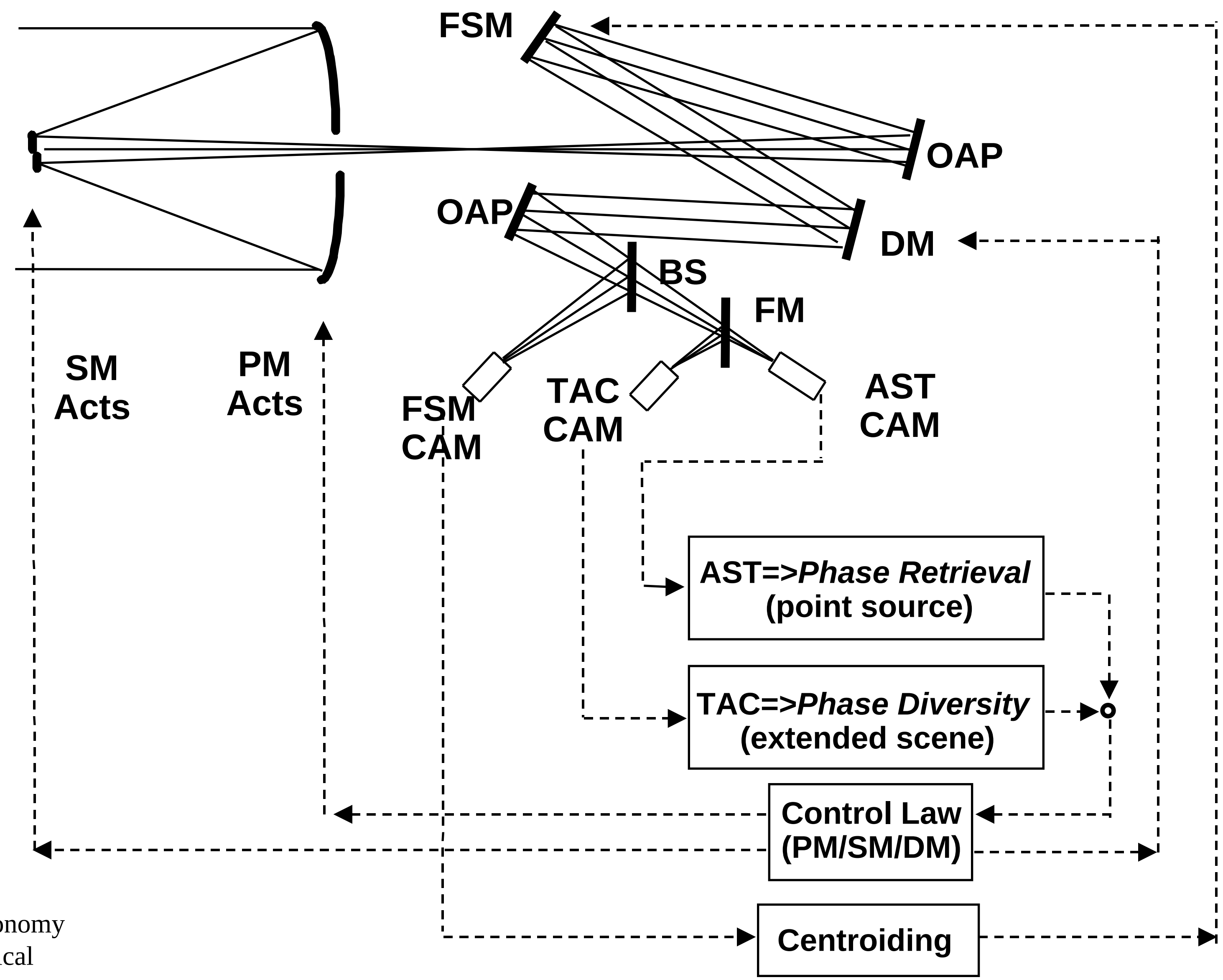
Example of Optical Control Loop



AST = Astronomy
TAC = Tactical

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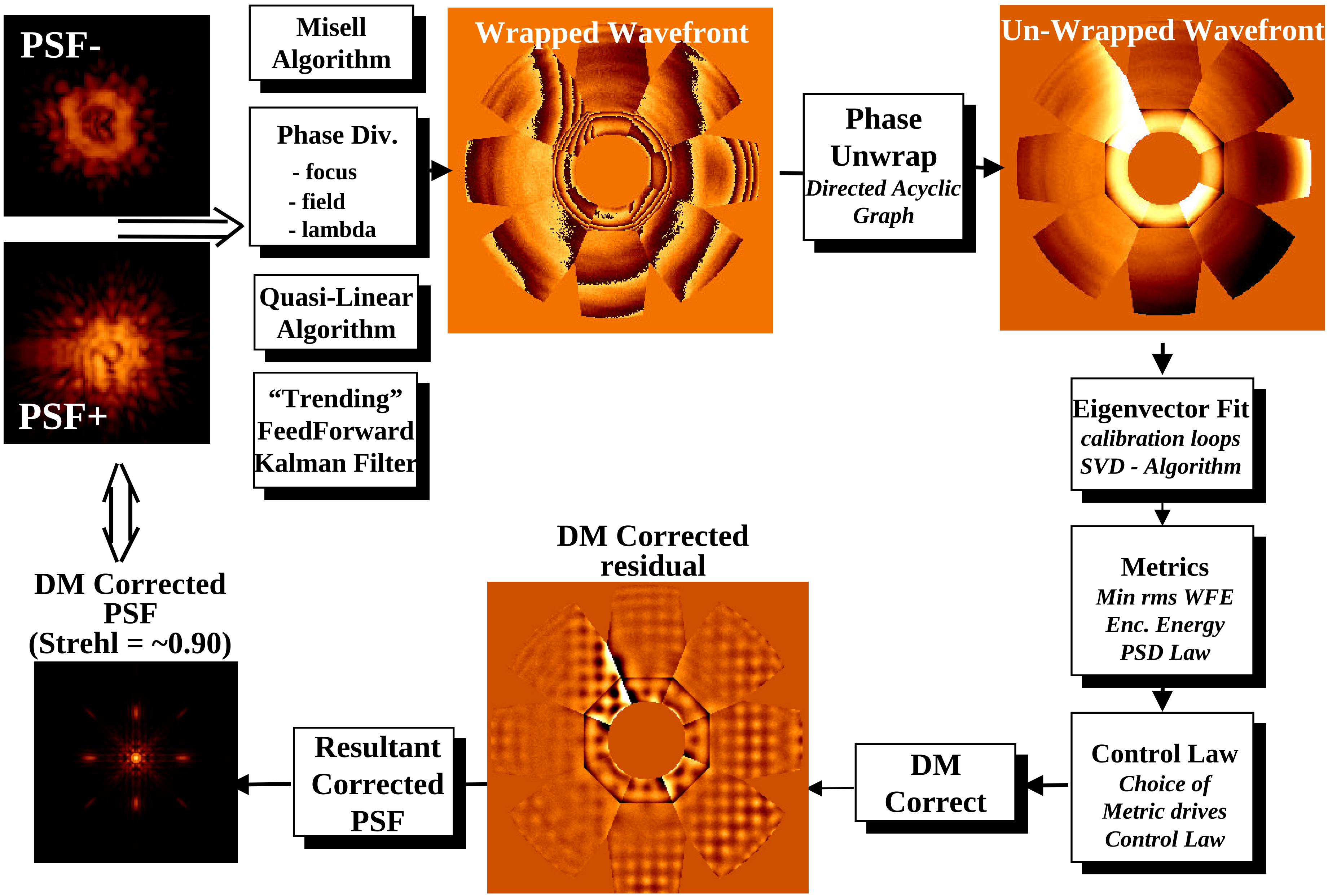
NGST Heirarchical Optical Control Loop



Statement of the Problem

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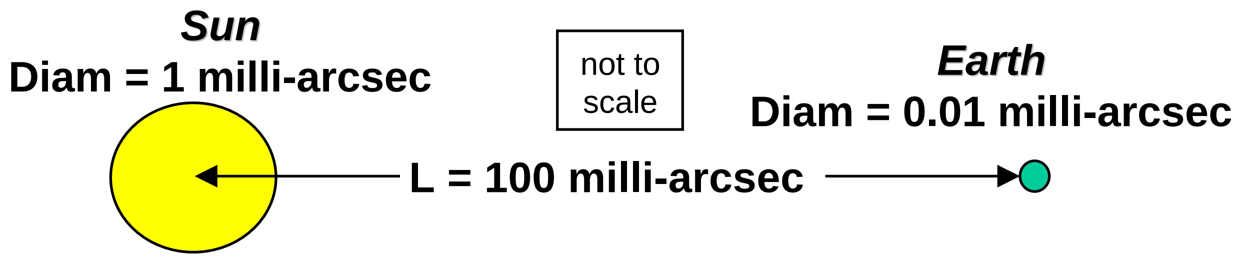
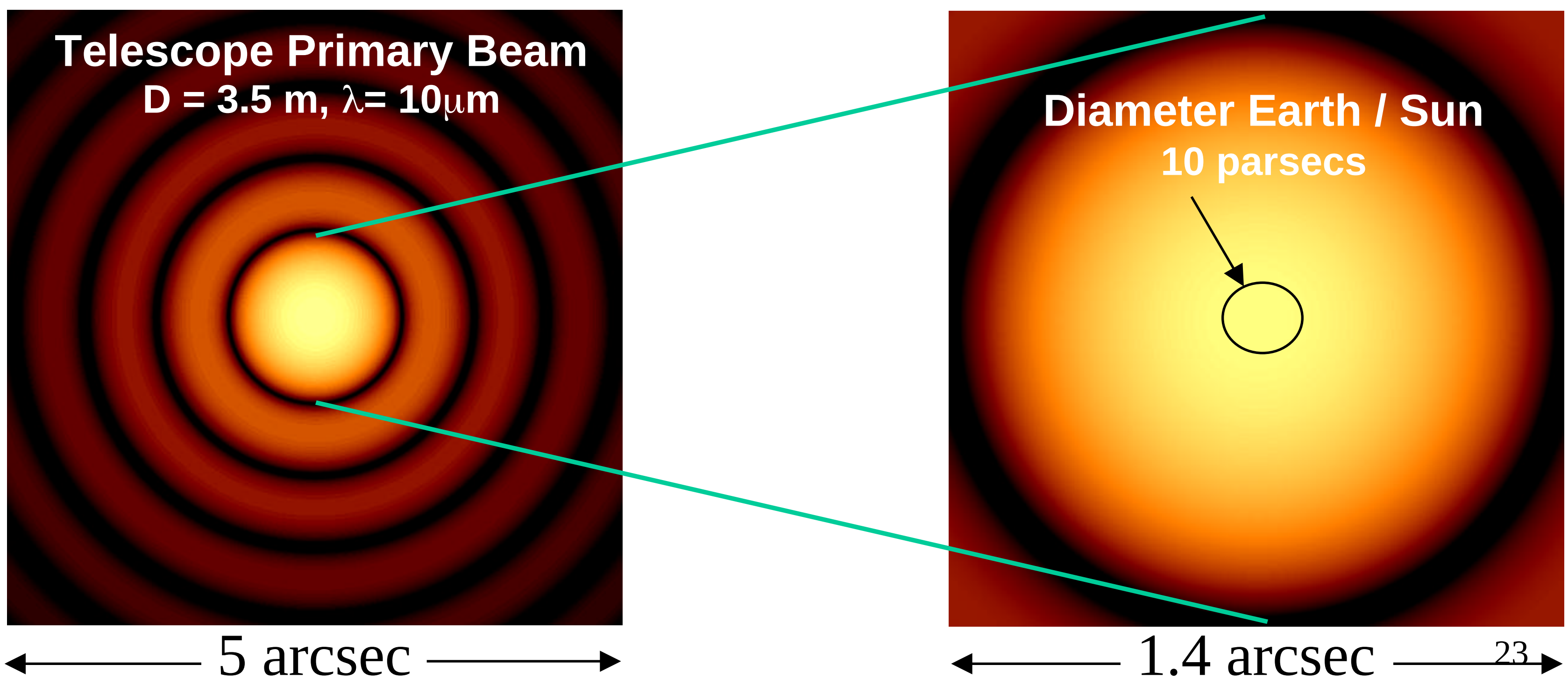


$$B_{sun} = 10^7 \, B_{Earth} \ @ \ \lambda = 10 \ \mu m$$



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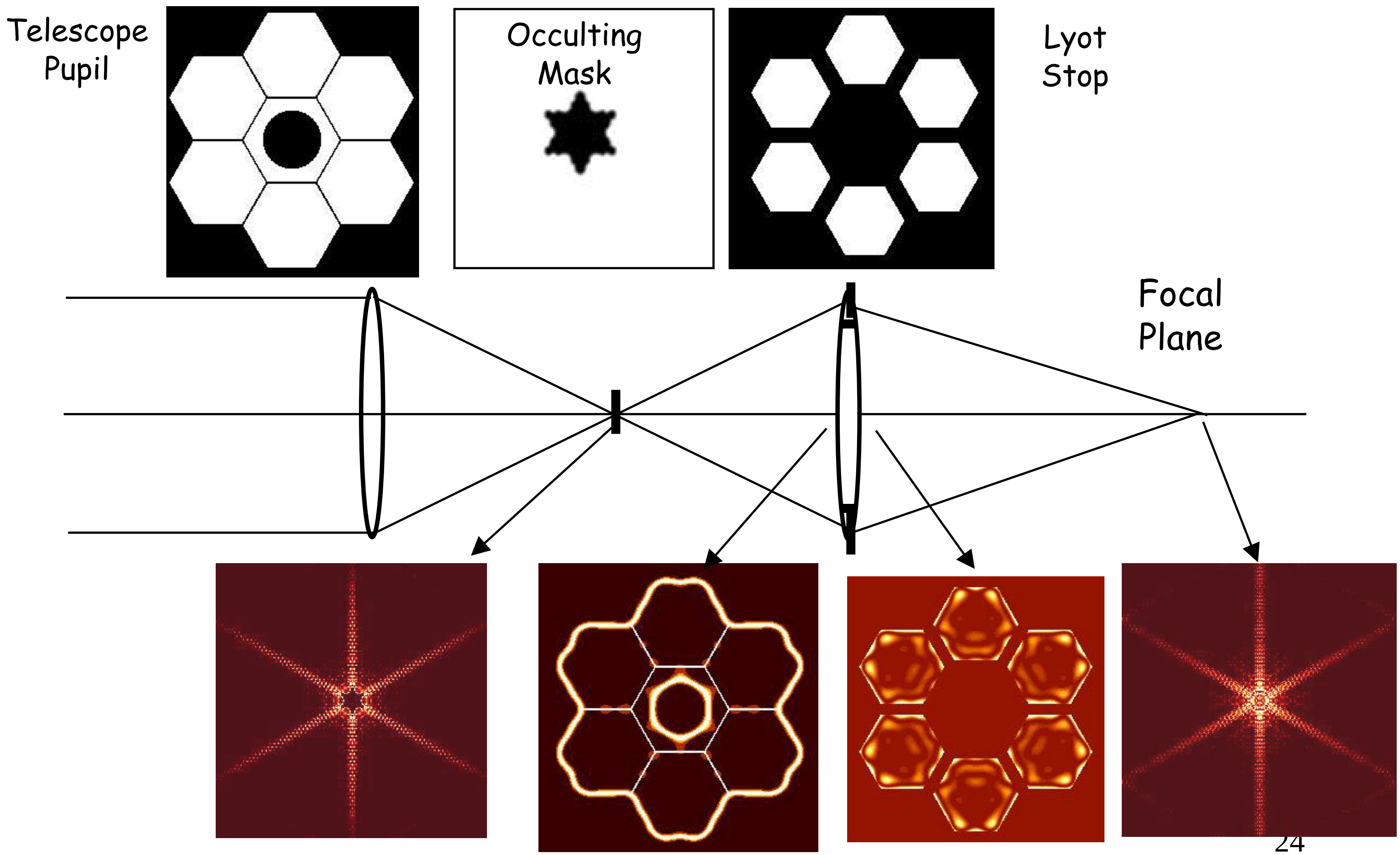
Principle of Lyot Coronagraph



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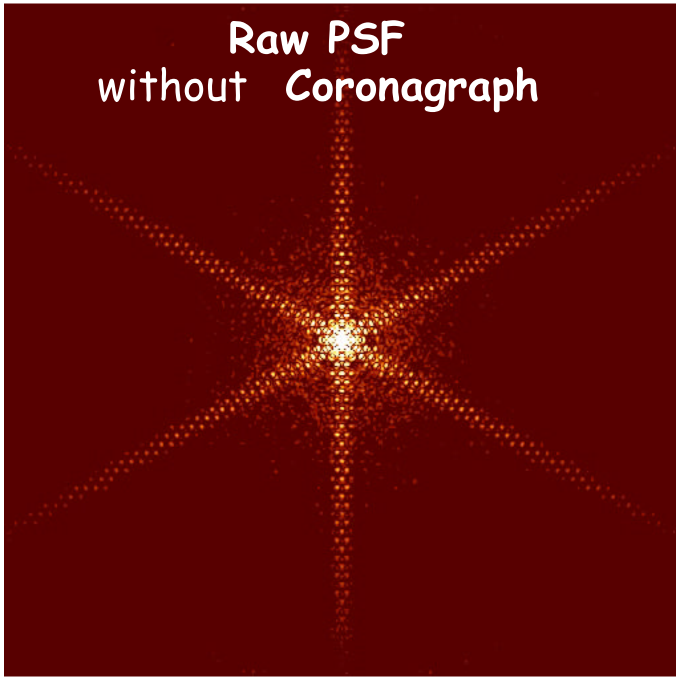

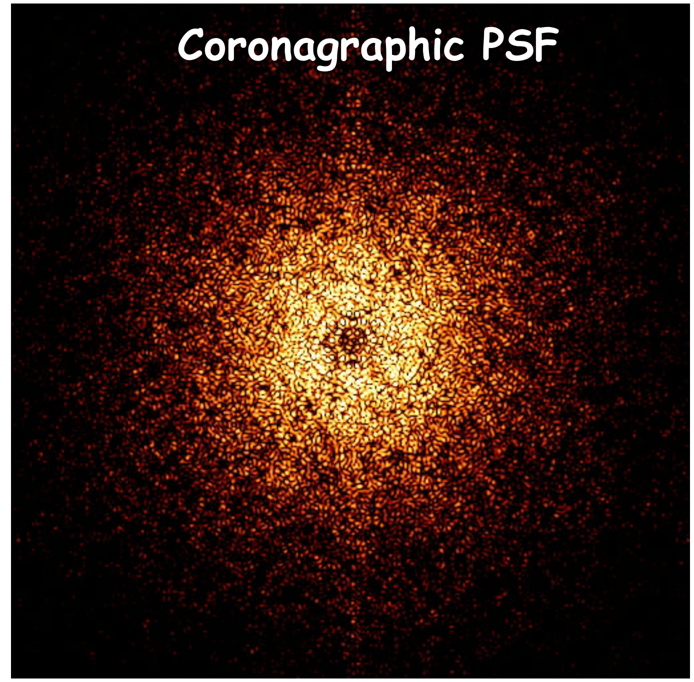

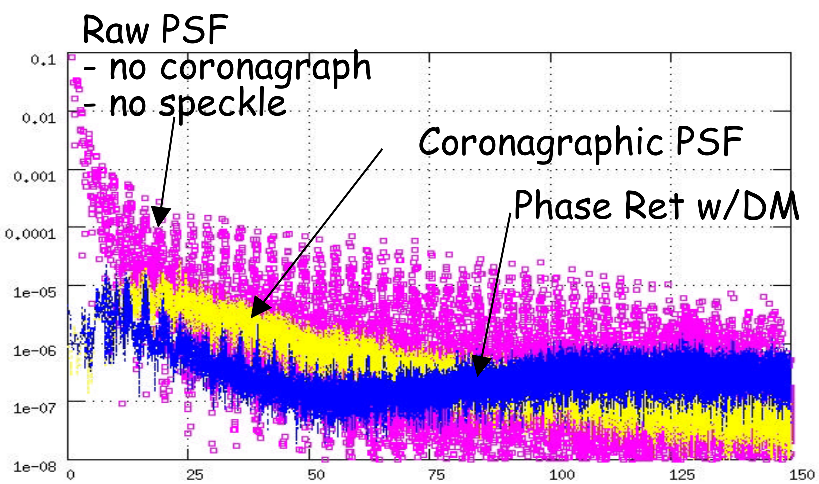

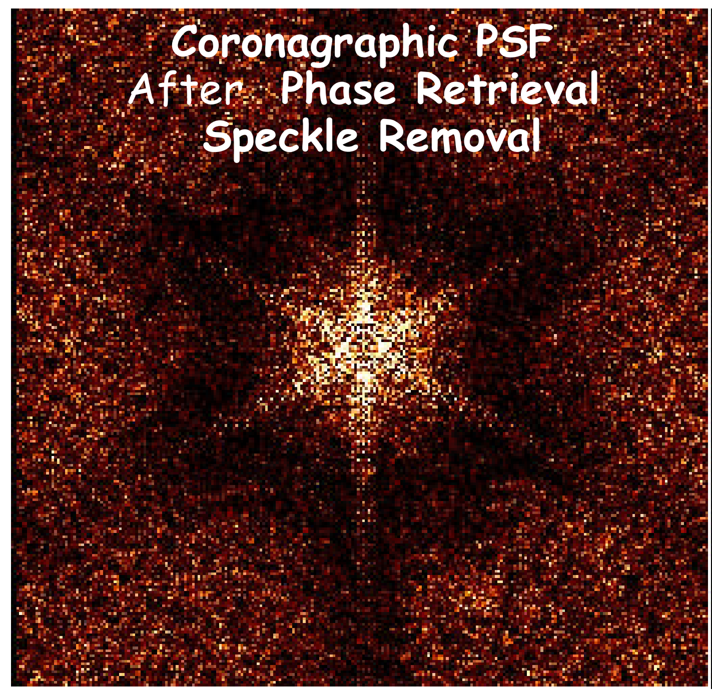

Apodized Square Aperture PSFs

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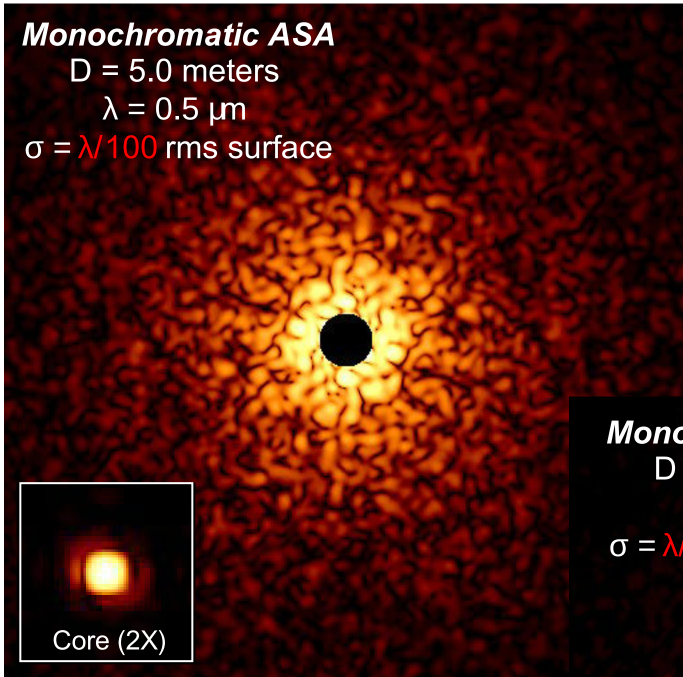

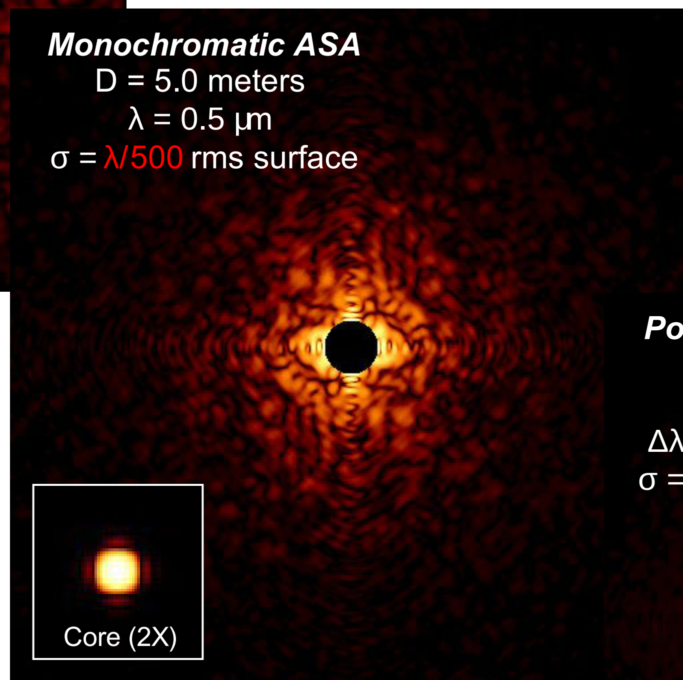

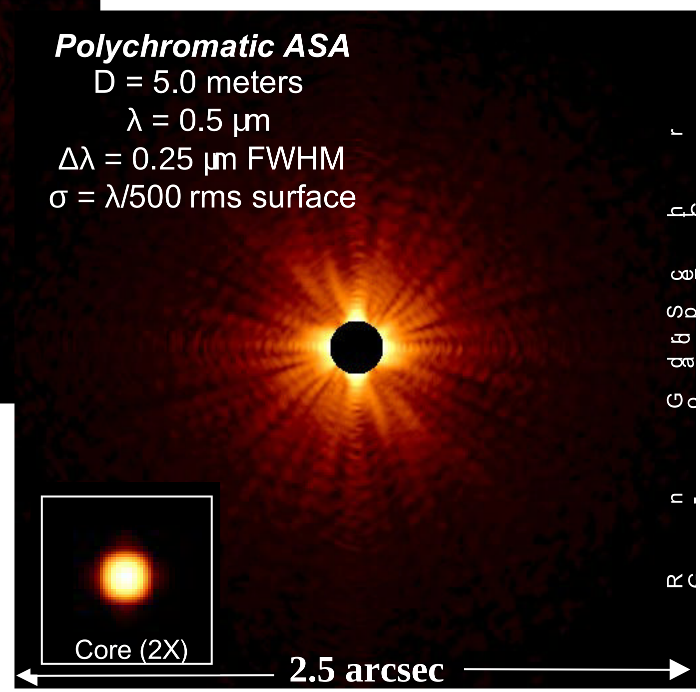

- Surface roughness

$$\text{Surface PSD} \sim \frac{A}{1+\left(f/f_0\right)^{3.55}}$$

- Sonine Apodized Square Ap

$$T(x,y) = \left[1-\left(\tfrac{2x}{D}\right)^2\right]^{\nu-1} \left[1-\left(\tfrac{2y}{D}\right)^2\right]^{\nu-1}$$

with ν=4

Purpose of *masked center*
is only for display purpose

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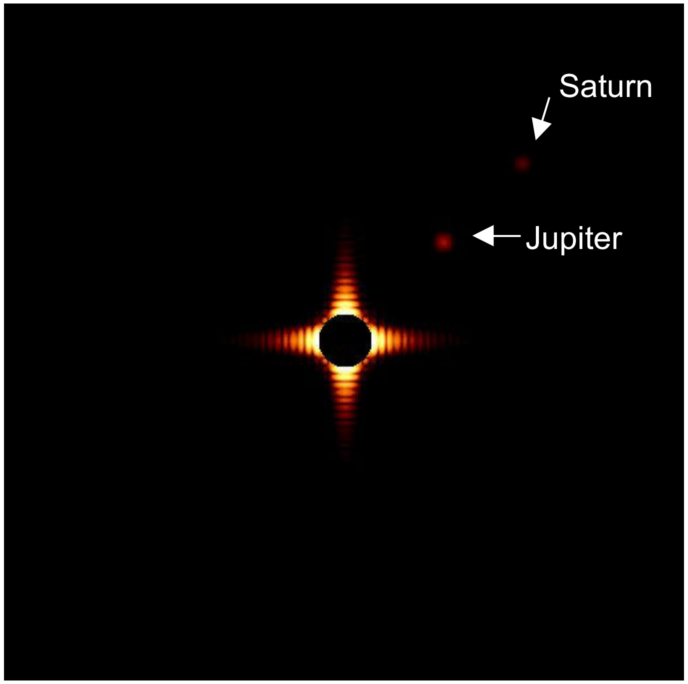
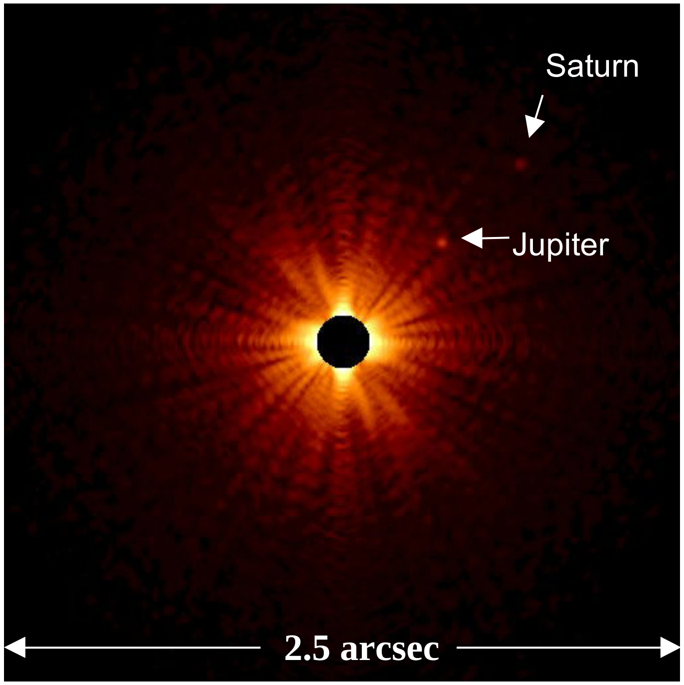
Apodized Square Aperture PSFs

Solar System Simulation

Our Solar System at 10 parsecs in visible light



Polychromatic ASA
D = 5.0 meters
λ = 0.5 µm
$\Delta\lambda$ = 0.25 µm FWHM
σ = No Wavefront error



Polychromatic ASA
D = 5.0 meters
λ = 0.5 µm
$\Delta\lambda$ = 0.25 µm FWHM
σ = λ/500 rms surface

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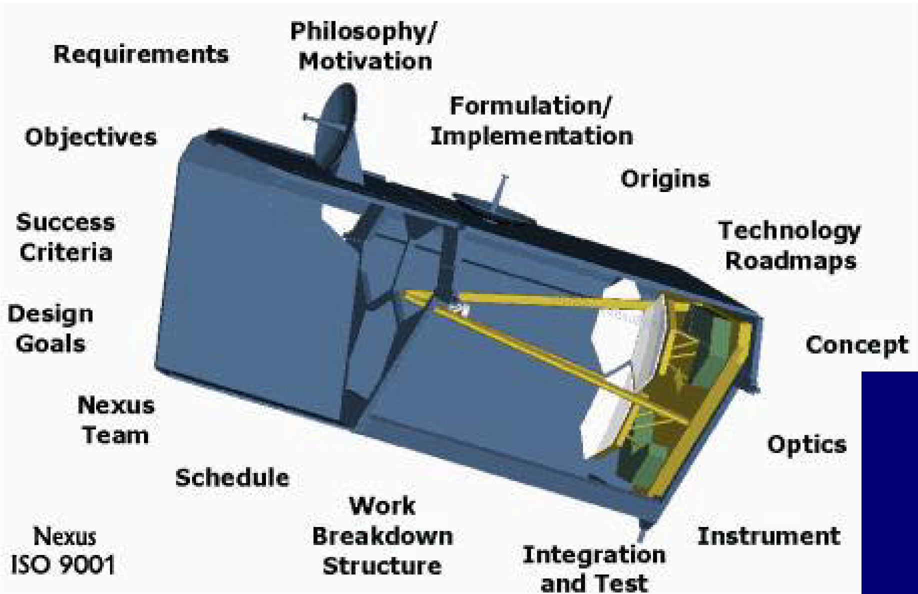
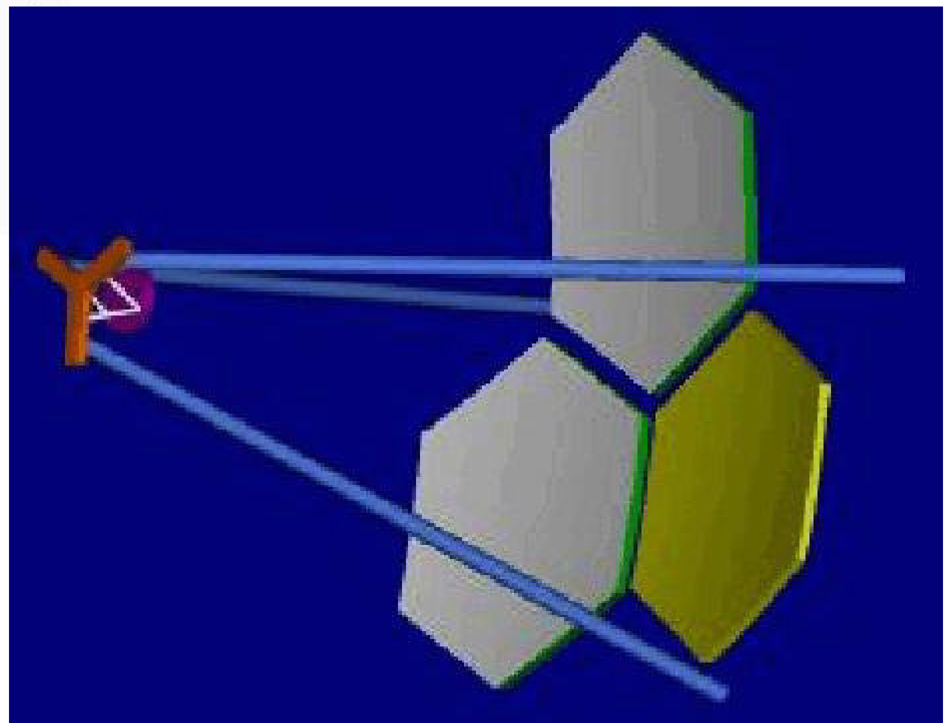
Horizon<=> Nexus Concept





Horizon
Phase Diverse Optical Control Loop

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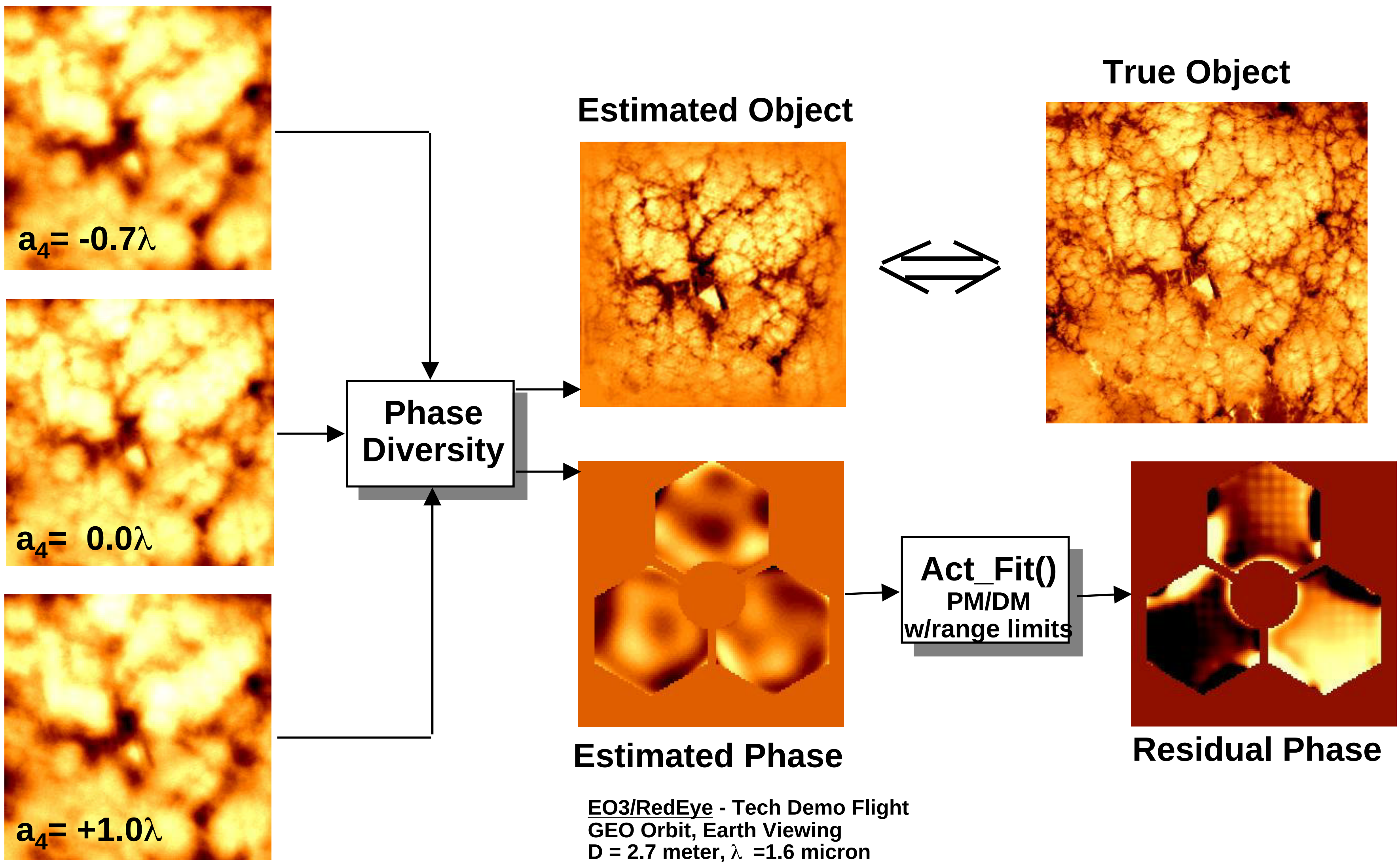

EO3/RedEye - Tech Demo Flight
GEO Orbit, Earth Viewing
D = 2.7 meter, λ =1.6 micron
3 Hex segments
PM/DM Acts

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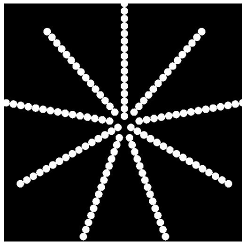
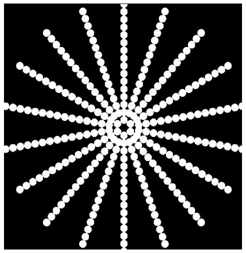
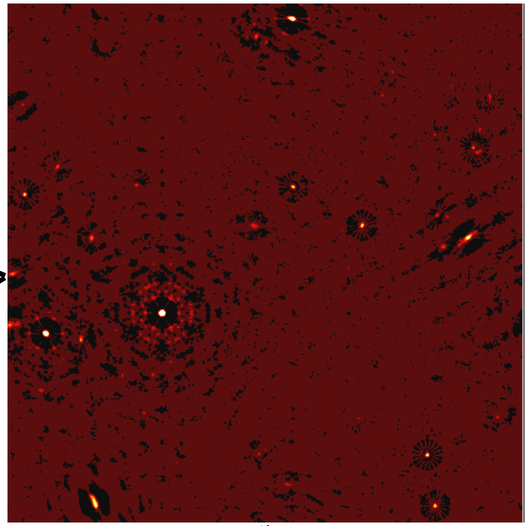
Synthetic Aperture Submillimeter Wave Imaging Simulations

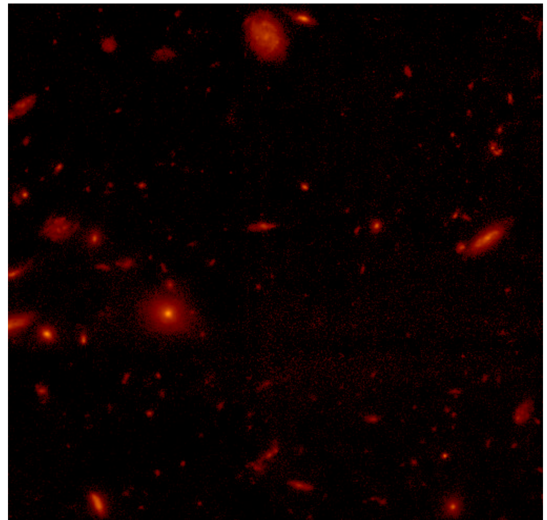
Aperture Postions





UV-Transfer Function

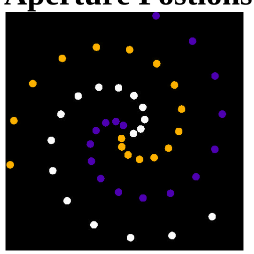


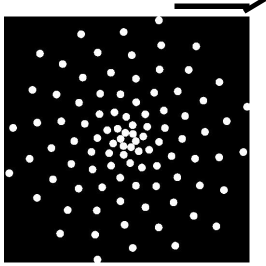


Dirty Maps

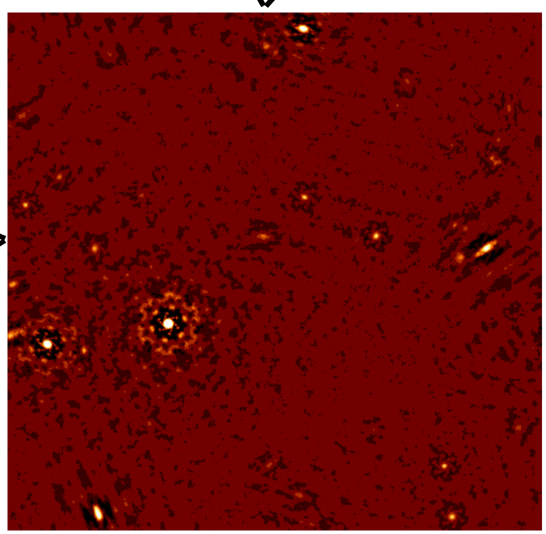
Maximum Entropy Cleaned Maps
Algorithm: R.Lyon,J.Hollis, J Dorband
ApJ, 478:658-662, 1997 April 1

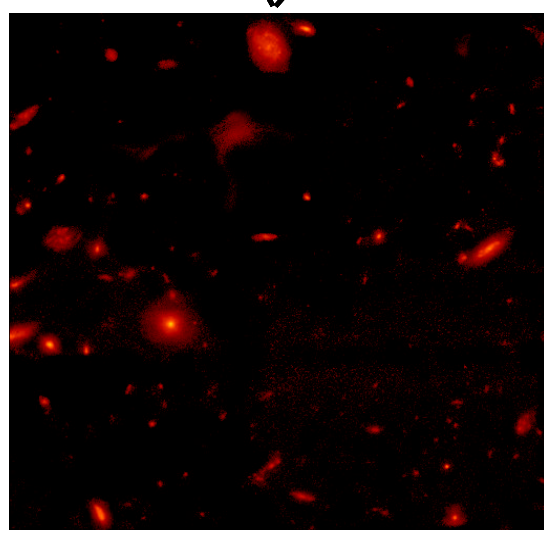
Aperture Postions

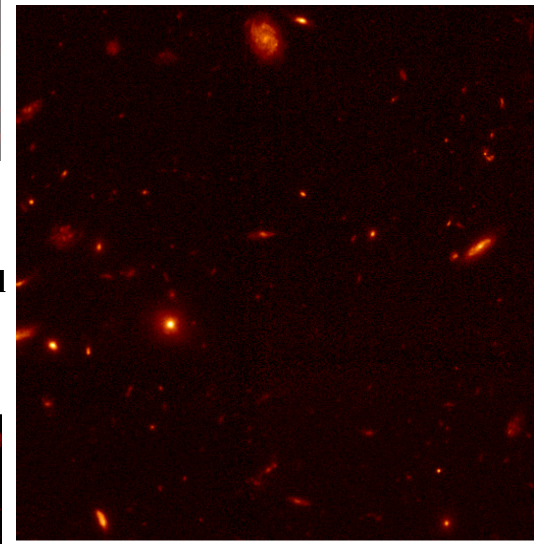




UV-Transfer Function







WF F606
Hubble Deep Field
0.0996 arcsec/pix, 1024x1024,
field = 102 arcsec
(log stretched)

u(max)=B(max)/λ= 512*9.8e-03=5cyc/sec
at λ= 50 um, B = ~ 52 meters
at λ= 100 um, B = ~104 meters
at λ= 200 um, B = ~208 meters
at λ= 400 um, B = ~416 meters

In Summary...

- Given motivation for Computational Optics at GSFC.
- Learned basic tools of forward optical modeling
- Seen a series of space applications at GSFC

For Further Reading...

- JD Jackson, "Classical Electrodynamics", 2nd edition, Wiley and Sons, 1975, pages 427-438.
  - Historical and rigourous diffraction theory.
- JW Goodman, "Introduction to Fourier Optics", McGraw-Hill, 1968.
  - Great book -> ties most aspects of Fourier optics together.
- JW Goodman, "Statistical Optics" , Wiley-Interscience, 1985
  - Coherence theory, Fourier optics, turbulence theory
- L. Mandel & E. Wolf, "Optical Coherence and Quantum Optics", Cambridge U. Press, 1995
- M. Born & E. Wolf, "Principles of Optics", Pergamon Press, 1984

Point of Contact

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Some of My Relevant References...

- R. G. Lyon, J. M. Hollis, J.E. Dorband, T.P. Murphy, "Extrapolating HST Lessons to NGST", Optics and Photonics News, Vol. 9, No. 7 (1998)
- R. G. Lyon, J. M. Hollis , J.E. Dorband, "A Maximum Entropy Method with A Priori Maximum Likelihood Constraints", Ap.J., 478, 658-662 (1997).
- R. G. Lyon, J.E. Dorband, J.M. Hollis, "Hubble Space Telescope Faint Object Camera Calculated Point Spread Functions", Applied Optics, 36, No. 8 (1997).
- R.G. Lyon, D.Y. Gezari, P. Nisenson, "Analysis of High Contrast Imaging Techniques for Space Based Direct Planetary Imaging", 198th Meeting of the American Astronomical Society, June 3-7, 2001, Pasadena CA
- Lyon, R.G., Solyar, G., Dorband, J.E., Ranawake, U.A., "Preliminary Phase Diverse Imaging Testbed Algorithms and Results", Workshop on Computational Optics and Imaging, NASA/GSFC, May 10,11,12, 2000